Physics 174 17 November, 2006
Homework 5 - Due 29 November

Name: _____

Kepler's Third Law and Orbital Resonances

Table: _____

In this assignment, we will explore how to use Kepler's Third Law to measure the mass of an object, and we also learn about orbital resonances.

1. The Mass of Jupiter

Kepler's Third Law relates the mass of an object to the periods and semi-major axes of any moons orbiting it. If we know two of these quantities, we can solve for the third. Jupiter has an extensive system of satellites. We can measure their orbital periods and distances and from this measure the mass of Jupiter.

In general, Kepler's Third Law takes the form:

$$p^2 = (4\pi^2/GM)\, a^3,$$

where p is the period, a is the semi-major axis, M is the mass of the central body, and G is the gravitational constant (6.67×10^{-11} N m^2 kg^{-1}). We can simplify this equation a little with a proper choice of units. For the orbits of moons around Jupiter, we can express the period p in days, the semi-major axis a in units of one million km (a gigameter, Gm!), and the mass M in solar masses. Then, Kepler's Third Law becomes:

$$p^2 = (C/M)\, a^3,$$

where

$$C = 0.0398\ M_{Sun}\ d^2\ Gm^{-3}.$$

a. Solve the above equation for M. What is the formula? M = _____.

b. Fill in the following table using the equation above.

Moon	a (Gm)	p (d)	M_J (M_{Sun})
Io	0.422	1.77	_____
Europa	0.671	3.55	_____
Ganymede	1.070	7.15	_____
Callisto	1.880	16.7	_____

c. What is the mass of Jupiter as a percentage of the mass of the Sun? _____

2. Orbital Resonances in the Asteroid Belt

Jupiter's mass dominates the Solar System within several AU of its orbit. The best example is the Asteroid Belt, which is full of material that might have formed into a planet had Jupiter not interfered.

If we express the period p in years, the semi-major axis a in AU, and set $M = 1\ M_{Sun}$ (since the asteroids are orbiting the Sun), Kepler's Third Law becomes very simple:

$$p^2 = a^3,$$

a. Solve this equation for p as a function of a. $p =$ _____.

b. The Kirkwood gaps are places in the Asteroid Belt which have been cleared by orbital resonances with Jupiter. For several gaps, the semi-major axis a is given below. Find the period of the orbit in that gap in years.

a (AU)	p (yr)	period ratio	resonance
2.064	_____	_____	_____
2.501	_____	_____	_____
2.824	_____	_____	_____
2.957	_____	_____	_____
3.277	_____	_____	_____

c. Jupiter's orbital period is 11.864 years. For each Kirkwood gap above, find the ratio $11.864/p$, and put your answer in the column for period ratio. In the final column, describe the resonance as 2:1, 3:2, etc.

Name: _____

3. Orbital Resonances in the Galilean System

Table: _____

Io, Europa, and Ganymede all show evidence of geologic activity (in the case of Io, it is pretty obvious). This activity has been tied to orbital resonances between the three moons. The question to answer is what those resonances are.

a. In the table below, divide the period of the moon by the period of Io and record the the answer in the column for period ratio.

Moon	a (Gm)	p (d)	period ratio	resonance
Io	0.422	1.77	1.00	1:1
Europa	0.671	3.55	_____	_____
Ganymede	1.070	7.15	_____	_____
Callisto	1.880	16.7	_____	_____

b. In the table above, round the number to the nearest tenths and write the result as a ratio in the last column (e.g. 3:1).

c. Which of the moons is farthest from a resonance? _____

d. Which of the moons is least geologically active? _____

e. What comments do you have on to your answers to (c) and (d) above?

4. Orbital Resonances around Saturn

The following table gives the semi-major axes and orbital periods of the larger moons of Saturn.

Moon	a (Gm)	p (d)
Mimas	0.186	0.94
Enceladus	0.238	1.37
Tethys	0.295	1.89
Dione	0.377	2.74
Rhea	0.527	4.52
Titan	1.220	16.0
Iapetus	3.560	79.3

The Cassini Division lies between the B and A rings, between 117,500 and 122,300 km (0.118-0.122 Gm). Its inner edge, which is the outer edge of the B ring, is maintained by an orbital resonance with one Saturn's moons. To determine which moon is responsible, we must first determine the orbital period at 0.118 Gm. Saturn has a mass of 2.86×10^{-4} M_{Sun}. Substituting this into Kepler's Third Law gives:

$$p^2 = K a^3,$$

where

$$K = 139.4 \text{ d}^2 \text{ Gm}^{-3}.$$

a. Solve this equation for p as a function of a. p = _____.

b. Can you identify the resonance?

The outer edge of the B ring has a _____ : _____ with the moon _____.

c. The Cassini mission has confirmed that Enceladus is geologically active. Does it have a resonance with any other moon? If so, identify the resonance.